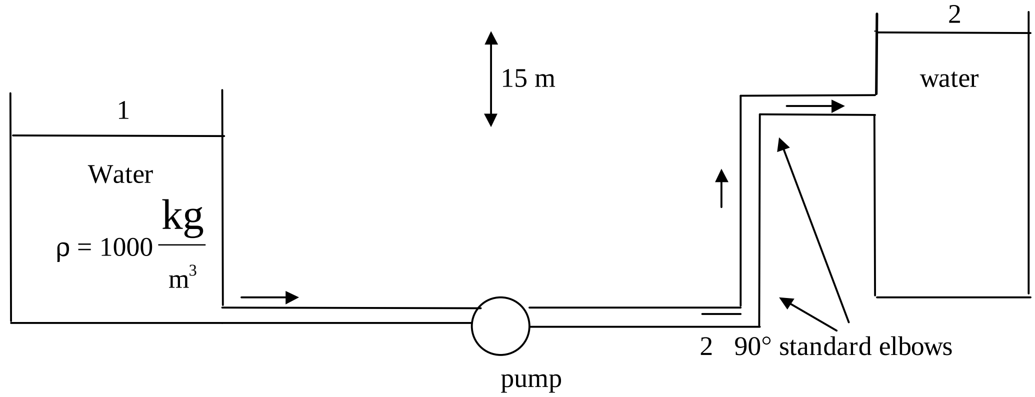
Example Problem on Frictional Loss in Pipes Calculations



μ water = 0.001005 Pa·s

pipe : wrought iron
length of pipe between tanks, L = 170 m
diameter (inner) of the pipe = 10·23 cm

mass flow rate = 18000 kg/h

Calculate the pump power to pump the water at the above rate.

$$\frac{P_1}{r} + \frac{V_1^2}{2a} + gz_1 + W_p - \frac{\Delta P_f}{r} = \frac{P_2}{r} + \frac{V_2^2}{2a} + gz_2$$

0

$V_1 \approx, V_2 \approx o$ assuming tanks with large diameter

$$\therefore W_p = g(z_2 - z_1) + \frac{\Delta P_f}{r}$$

$z_2 - z_1 = 15 m.$ ....**I**

$$\text{Losses} \rightarrow \frac{\Delta P_f}{r} = \frac{\Delta P_{f\,L}}{r} + 2\,\frac{\Delta P_{f\,Le}}{r} + \frac{\Delta P_{f\,cont}}{r} + \frac{\Delta P_{f\,exp.}}{r}$$

Total; 1)due to pipe length (friction loss); 2) due to 2 elbows 90°; 3)due to sudden contraction in 1st tank; 4) due to sudden expansion in 2nd tank

To calculate all losses, first calculate Reynolds number

$$Q\left(m^3/s\right)=\frac{\dot{m}}{r}=\frac{18000}{(3600)(1000)}=0\cdot005\ m^3/s$$

$$\overline{V}=\frac{Q}{\frac{pD^2}{4}}=\frac{0\cdot005}{\frac{p(0\cdot1023)^2}{4}}=0\cdot6083\ m/s$$

$$\therefore\ Re\ =\frac{r\overline{V}D}{m}=\frac{(1000)(0\cdot6083)(0\cdot1023)}{0\cdot001005}=61921>2100$$

$\therefore$ Flow is turbulent.

For wrought iron,

$e=45\cdot7x10^{-6}\,m$

from Figure 2-10.3, Page 88 in the textbook

$$\therefore\ \text{relative roughness}\ =\ \frac{e}{D}=\frac{45\cdot7x10^{-6}}{0\cdot1023}\cong0\cdot00045$$

$\therefore$ for $e/D=0\cdot00045$, $Re=61921$, $f\cong0\cdot0052$

Now we calculate all the losses.

$$1)\ \frac{\Delta P_{fL}}{r}=\frac{2fL\overline{V}^2}{D}=\frac{(2)(0\cdot0052)(170)(0\cdot6083)^2}{0\cdot1023}$$

$$=0\cdot654/0\cdot1023=6\cdot395\ J/kg$$

2) $\dfrac{\Delta P_{f\,L_e}}{r}$: loss due to one 90° standard elbow

$$\frac{\Delta P_{f\,L_e}}{r} = \frac{2f\,L_e\,\overline{V}^2}{D}$$

Le: equivalent length for standard 90° elbow

From Table 2·10-1, pp. 93, $\dfrac{Le}{D} = 35 \;\therefore\; Le = 35D$

$$\therefore \frac{\Delta P_{f\,Le}}{r} = \frac{2(f)(35)(D)\overline{V}^2}{D} = \frac{70f\,V^2}{1}$$

$$= (70)(0\cdot 0052)(0\cdot 6083)^2 = 0\cdot 134 \text{ J/kg}$$

3) $\dfrac{\Delta P_{f\,cont}}{r}$: energy loss due to sudden contraction at the tank 1

From equation 2·10-16, pp. 93,

$$\frac{\Delta P_{f\,cont}}{r} = K_f \frac{\overline{V}^2}{2}$$

We assume tank diameter to be very large as compared to the pipe diameter $\therefore \quad D_2 / D_1 \ll 1 \approx 0$

$$\therefore k_f \cong 0\cdot 55(1-0) \cong 0\cdot 55$$

$$\therefore \frac{\Delta P_{f\,cont}}{r} = (0\cdot 55)\frac{(0\cdot 6083)^2}{2} = 0\cdot 102 \text{ J/kg}$$

4) $\dfrac{\Delta P_{f\,exp}}{r}$: loss due to sudden expansion in tank 2

from eqn. 2·10-15, pp. 93,

$$\frac{\Delta P_{f\,exp}}{r} = \frac{\overline{V}^2}{2}\left(1 - A_2^2 / A_1^2\right)^2$$

again, assuming $A_2 << A_1$ or $A_2 / A_1 \approx 0$

$$\frac{\Delta P_{f\,exp}}{r} \cong \frac{\overline{V}^2}{2} = \frac{(0\cdot 6083)^2}{2} = 0\cdot 185 \text{ J/kg}$$

$$\therefore \frac{\Delta P_f}{r} = 6\cdot 395 + 2(0\cdot 134) + 0\cdot 102 + 0\cdot 185$$

total = 6·95 J/kg

Substituting all values in **I**

$$W_p = 9\cdot 81(15) + 6\cdot 95$$
$$= 154.1 \text{ J/kg}$$

$$\therefore \text{pump power} = \left(\dot{m}\right)(W_p) = \left(\frac{18000}{3600}\right)(154) = 770\cdot 3$$

(kg/s) (J/kg) (Watts)

If the pump is 90% efficient, then the actual pump power needed would be $\dfrac{770\cdot 3}{0\cdot 90} = 855\cdot 89$ Watts